

Solving the corner-to-corner resistance of a cube with arbitrary resistances.

The rightmost node has a voltage of zero.

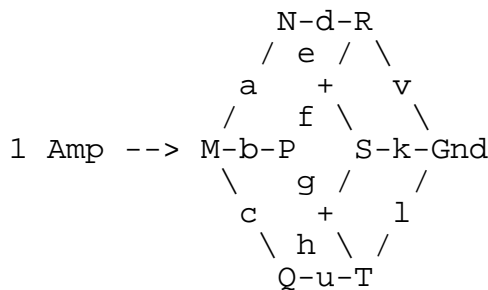
M is the voltage on the leftmost node. There is 1 amp flowing through the whole circuit, so M is also the final (resistance) answer in ohms.

Letters A, B, C, D, E, F, G, H, U, V, K, L are the resistances (sorry for the illogical layout of letters, but I and J were interpreted as complex numbers by Maple).

Letters M, N, P, Q, R, S, and T are the voltages at the various nodes.

Lower-case letters are the currents flowing through each resistor. I pre-substituted $c = (1 - a - b)$ to help Maple along a bit.

The formulae are simply of the form (voltage at one end of each resistor = value of resistor * current through resistor + voltage at other end of resistor), followed by Kirchoff's Current Laws. The structure of the cube is as follows, in bad ASCII art:



This step isn't intended to be readable, skip to the last two pages to be able to see anything remotely readable by humans.

```

> eliminate( {R = V*v + 0,
S = K*k + 0,
T = L*l + 0,
N = D*d + R,
N = E*e + S,
P = F*f + R,
P = G*g + T,
Q = H*h + S,
Q = U*u + T,
M = A*a + N,
M = B*b + P,
M = C*(1-a-b) + Q,
v + k + l = 1,
d + f = v,
e + h = k,
g + u = l,
a = d + e,
b = f + g,
(1-a-b) = h + u}, {M, a, b, N, P, Q, d, e, f, g, h, u, R, S, T, v, k, l})[1][7];
M = (AUGFCV + KBAECF + KBEGDC + KBGVEC + HVB UAF + KBHDCV
  
```

(1)

+KBUVCD+HDBUAF+AHGFCE+AHEUGF+UGFCED
+HDACGV+KHDUGF+KEGDCV+KGEDFC+KHDUVG
+KUDBVE+KUEDFB+KUEGVF+HVBACF+LHAGVF
+AUGFCE+HGDFAC+HEVAUG+HDUVAG+BAUGCD
+BHVAUG+KUEDGV+KUEGBV+KUEGDF+KUEGBD
+KAHGFC+KAHUGF+KHVAUG+KBHVUF+KUVCDG
+KHGVFC+KBHVCF+KBHDUF+KBHDUV+KAUDGV
+LHDFBE+LHEGBV+HDBACF+KAUEGB+KAUDGF
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 +HVUAF+HDUAF+KBUEV+HVEAU+HVACF+KBUCV
 +LACEF+LACEV+LKACF+AHEUG+KBHCV)

Now let's try making all groups of parallel resistors have equal values:

> *simplify(subs(A=x, B=y, C=z, D=y, E=z, F=x, G=z, H=x, U=y, V=z, K=y, L=x, %))*;

$$M = \frac{1}{4} \frac{zy^2 + xy^2 + z^2y + 4xzy + x^2y + xz^2 + x^2z}{yz + yx + xz} \quad (2)$$

Note how each resistance term reaches a second power on the numerator, but only the first power on

the denominator. This means that as we take any axis's resistance to infinity, the total resistance will go to infinity as well.

Now let's try making the x and y resistance equal too:

```
> simplify(subs(x=y, %));
```

$$M = \frac{1}{2} \frac{y^2 + 3 y z + z^2}{y + 2 z} \quad (3)$$

And now let's make all the resistors 1 kilohm:

```
> subs(x=1000, y=1000, z=1000, %);
```

$$M = \frac{2500}{3} \quad (4)$$

```
> fsolve(%);
```

$$833.3333333 \quad (5)$$

```
>
```