Three-Corner-Hat Allan deviation of Voltage Standards

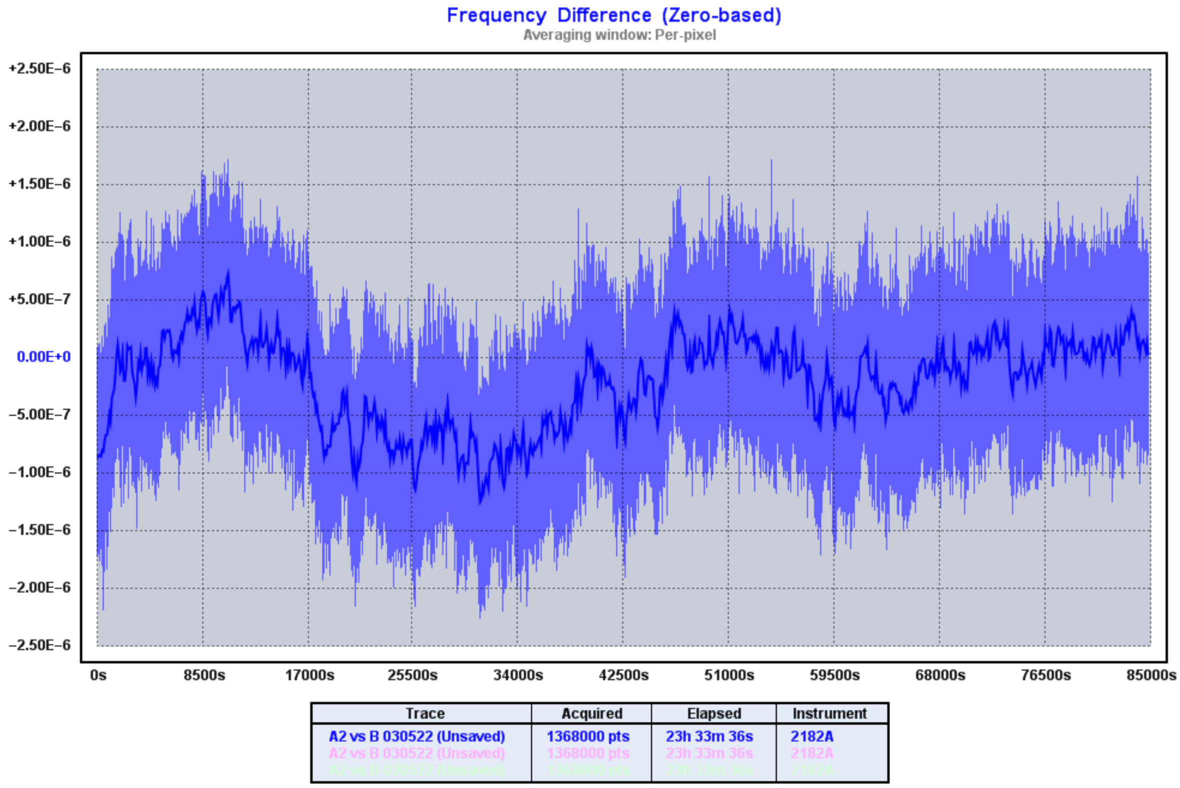
When measuring a DC voltage like the voltage of a Voltage Standard with a high resolution DMM, the received result is never stable and shows always some variation around the average – the noise. Now it would be desirable to find the origin of the instability. The noise-level is the RMS value of two instruments, the Voltage Standard and the Voltmeter. If the noise level of one instrument is known, the noise contribution of the other can be calculated.

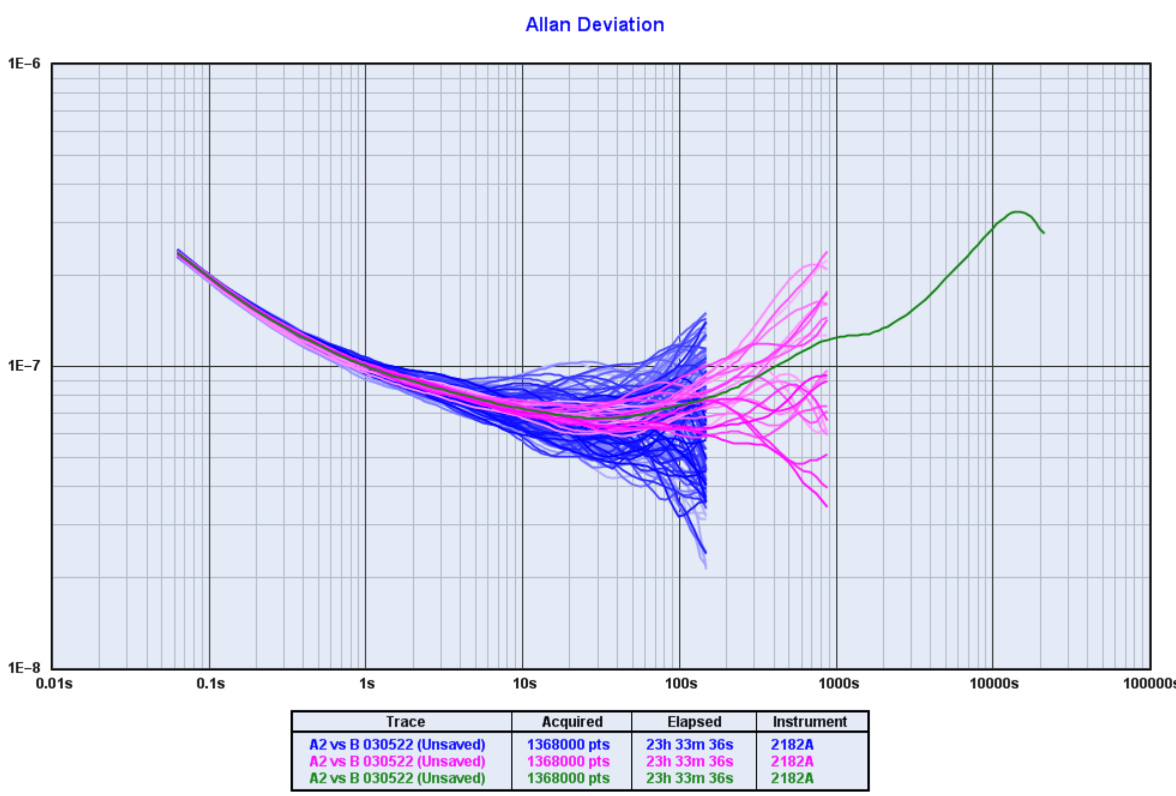
Some people use the standard deviation of the measurements with fixed sample rate to determine the noise-level. This method works well for mostly white noise signals like a short at the input of a DMM but it reaches its limit if you measure on a 1/f noise source like a voltage standard. To get a stable and repeatable standard deviation, you have to measure over a long time and longtime variations of your source will be the reason that your standard deviation is rising with time.

I am using the Allan Deviation of the differential of the noise level measured to determine the noise level. More data means better statistics.

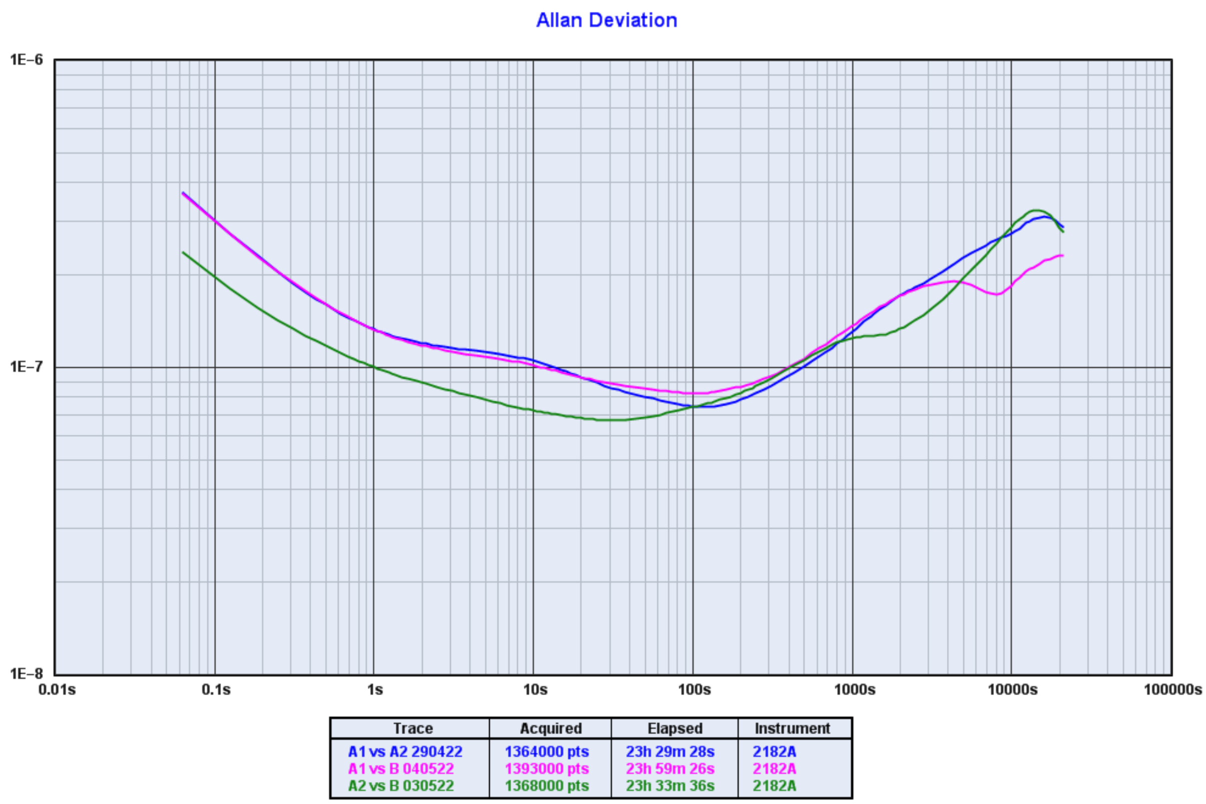
The next two pictures show a measurement of the differential voltage between a Fluke 732A and a Fluke 732B voltage standard measured with a Keithley 2182A Nanovoltmeter at a sample rate of 62ms over 23 hours and the related overlapping Allan Deviation. The used program is “Timelab” from John Miles. To show the stability of the Allan Deviation depending of the measuring time, the original 23h data (green line) is divided into 23 time intervals of around 1h (violet lines) and 141 time intervals of 10minutes (in blue). Y-axis is in Vrms.

The measured noise level is the RMS of three sources, the F32A#2, F732B and Keithley 2182A. The differential voltage between the Voltage References is in the order of some tenth of microvolts. The noise level of the K2182A at this voltage was assumed to be about the same as that of a shorted input which is everywhere more than ten times less. Therefore the contribution of the K2182A is neglected.


Frequency Difference (Zero-based)
Averaging window: Per-pixel
+2.50E−6
+2.00E−6
+1.50E−6
+1.00E−6
+5.00E−7
0.00E+0
−5.00E−7
−1.00E−6
−1.50E−6
−2.00E−6
−2.50E−6
0s
8500s
17000s
25500s
34000s
42500s
51000s
59500s
68000s
76500s
85000s
Trace
Acquired
Elapsed
Instrument
A2 vs B 030522 (Unsaved)
1368000 pts
23h 33m 36s
2182A
A2 vs B 030522 (Unsaved)
1368000 pts
23h 33m 36s
2182A


Allan Deviation
1E−6
1E−7
1E−8
0.01s
0.1s
1s
10s
100s
1000s
10000s
100000s
Trace
Acquired
Elapsed
Instrument
A2 vs B 030522 (Unsaved)
1368000 pts
23h 33m 36s
2182A
A2 vs B 030522 (Unsaved)
1368000 pts
23h 33m 36s
2182A
A2 vs B 030522 (Unsaved)
1368000 pts
23h 33m 36s
2182A

If both Voltage Standards would have identical noise, the deviation of each individual could be corrected by dividing the measured value by square root 2. To determine if both have the same noise, a third Voltage Reference is needed and three pairs of measurement have to be made. Such a measurement using Fluke 732A#1, Fluke 732A#2 and Fluke 732B are shown in the next plot.



Timelab has the feature to calculate the individual noise curves out of these three ADEV measurements by using the 3-cornered hat method.

Requiements:

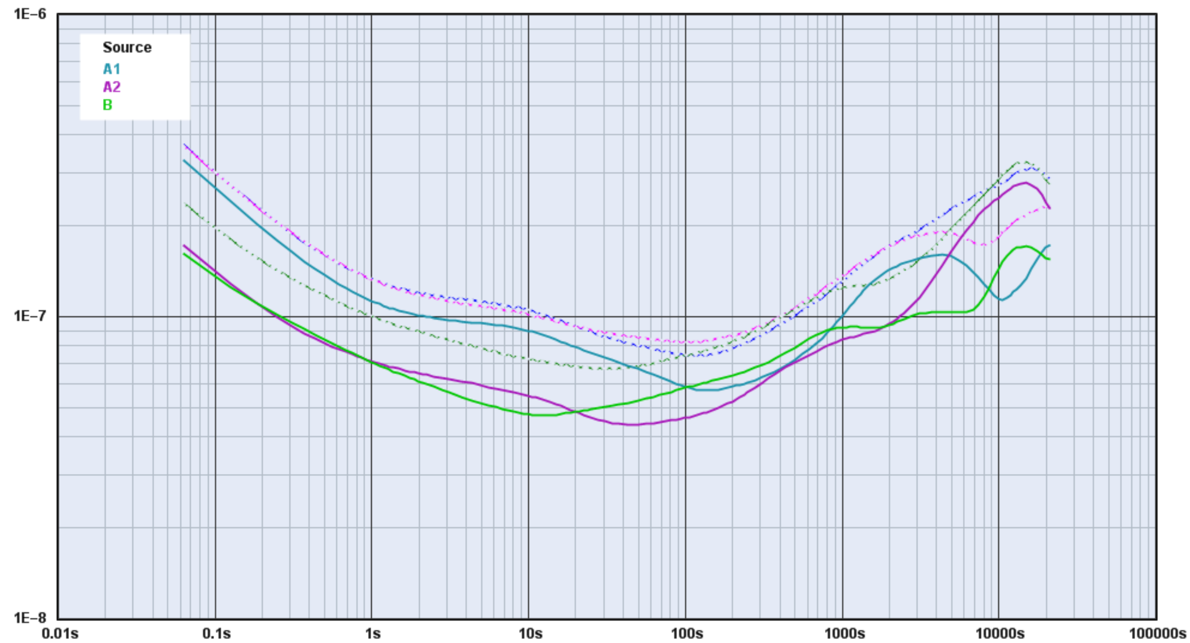
The three noise sources should have similar stability

The noise sources should be independent

The measurement of the three pairs should be made with the same sample rate.

Preferable, the measurements should be made at the same time. –I cannot fulfill this requirement, but the ADEV measurements of my Voltage Standards are very reproducible year for year.

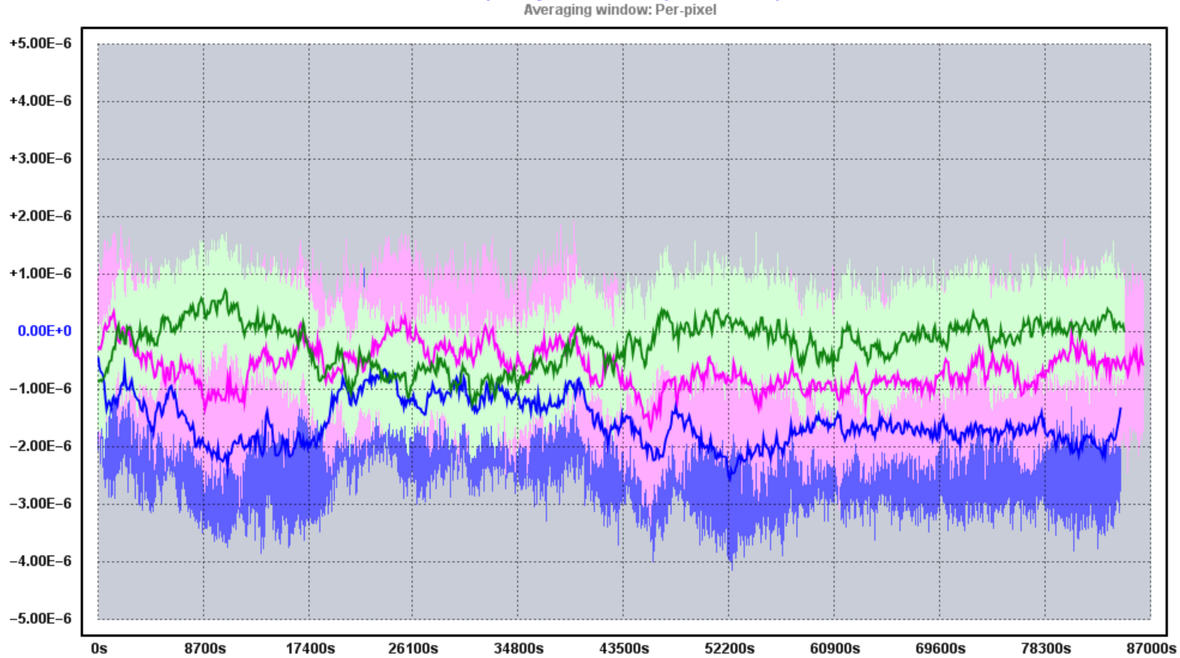
Allan Deviation (3-cornered hat)



Trace	Acquired	Elapsed	Instrument
A1 vs A2 290422	1364000 pts	23h 29m 28s	2182A
A1 vs B 040522	1393000 pts	23h 59m 26s	2182A
A2 vs B 030522	1368000 pts	23h 33m 36s	2182A

Frequency Difference (Zero-based)

Averaging window: Per-pixel

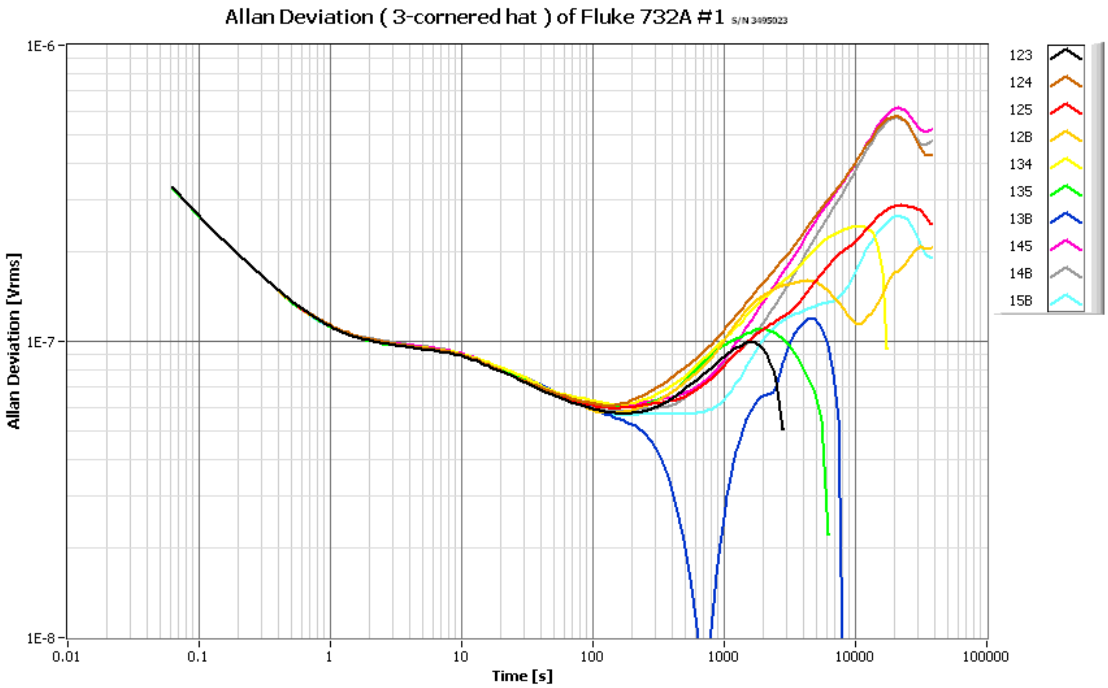


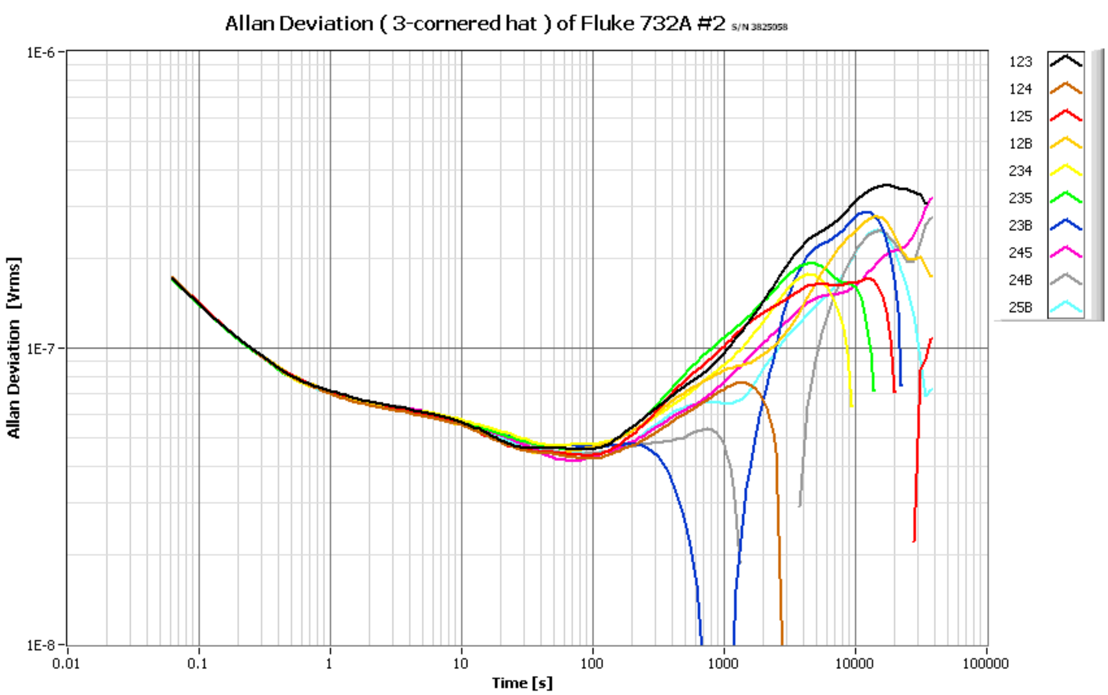
Trace	Acquired	Elapsed	Instrument
A1 vs A2 290422	1364000 pts	23h 29m 28s	2182A
A1 vs B 040522	1393000 pts	23h 59m 26s	2182A
A2 vs B 030522	1368000 pts	23h 33m 36s	2182A

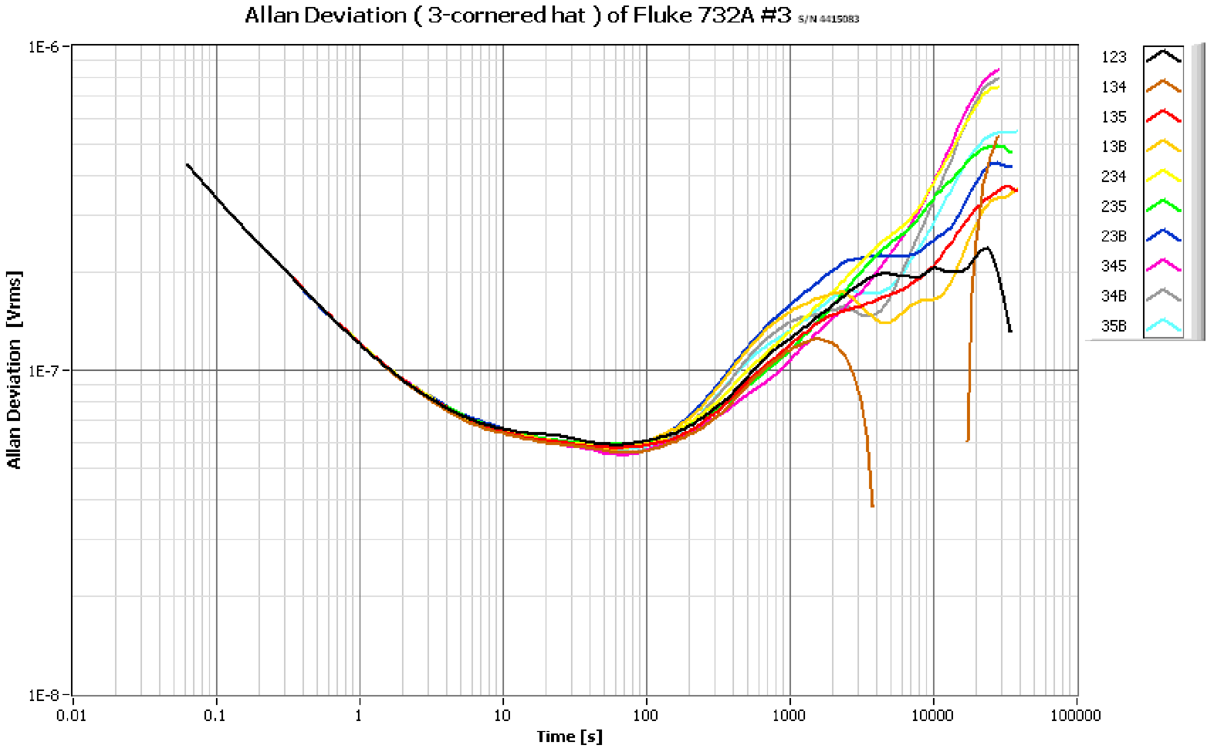
There is no similar measurement published on the internet and there could be a systematic error when measuring always between the same instruments.

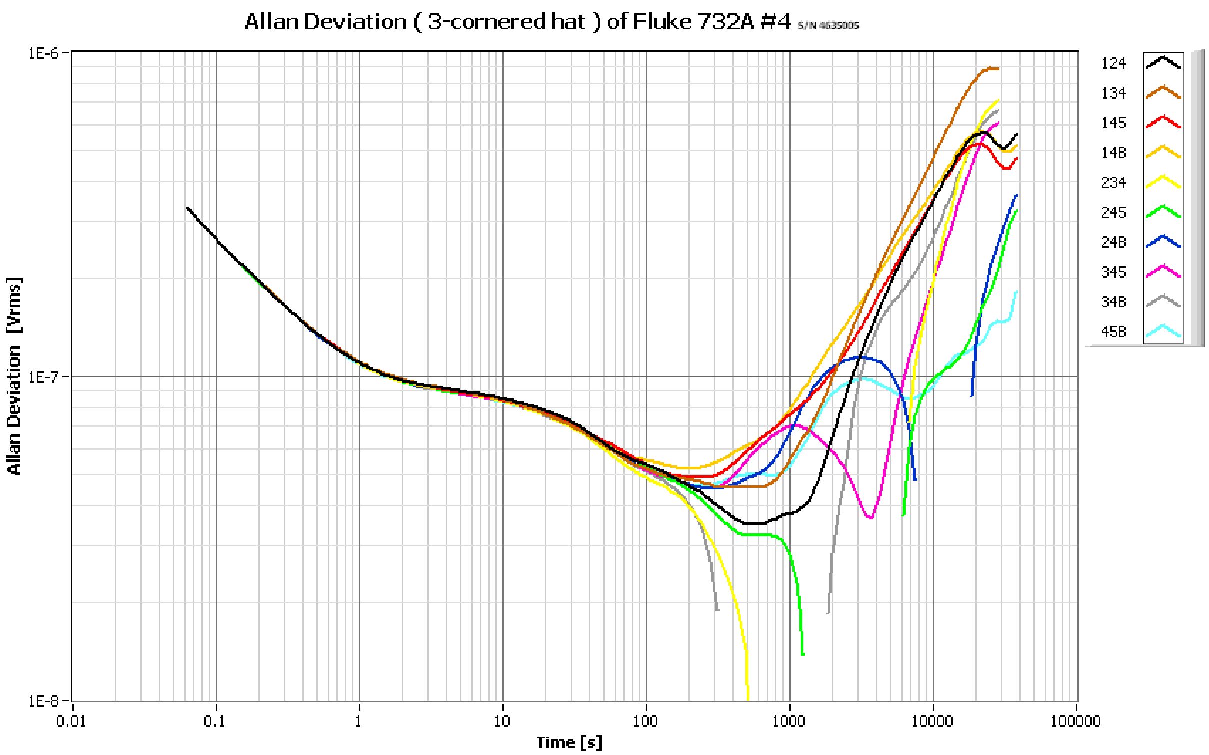
This year I have made measurements between six Voltage Standards – fife Fluke732A and one Fluke732B. This gives 15 different pairs and 20 different triples for 3-cornered hat calculations.

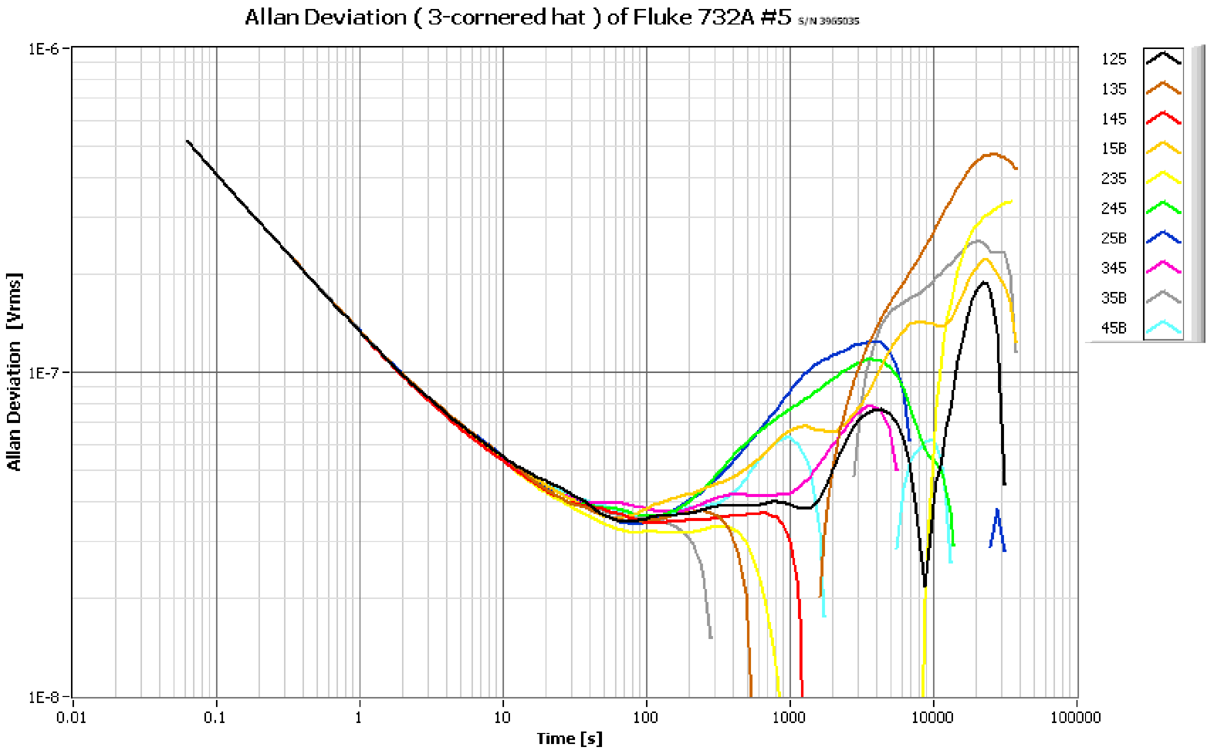
Therefore there are 10 3-cornered hat Allan Deviation solutions for each Voltage Standard.

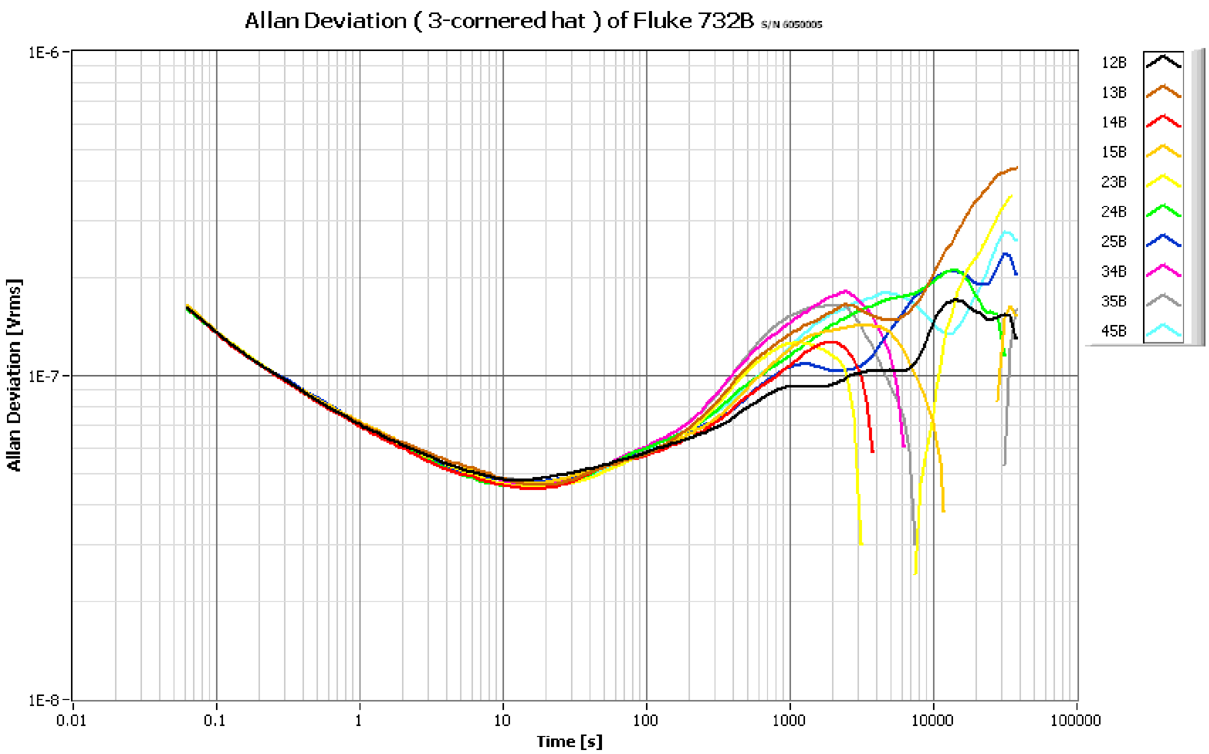





Allan Deviation (3-cornered hat) of Fluke 732A #3 S/N 4415083
Allan Deviation [Vrms]
1E-6
1E-7
1E-8
0.01
0.1
1
10
100
1000
10000
100000
Time [s]
123
134
135
13B
234
235
23B
345
34B
35B

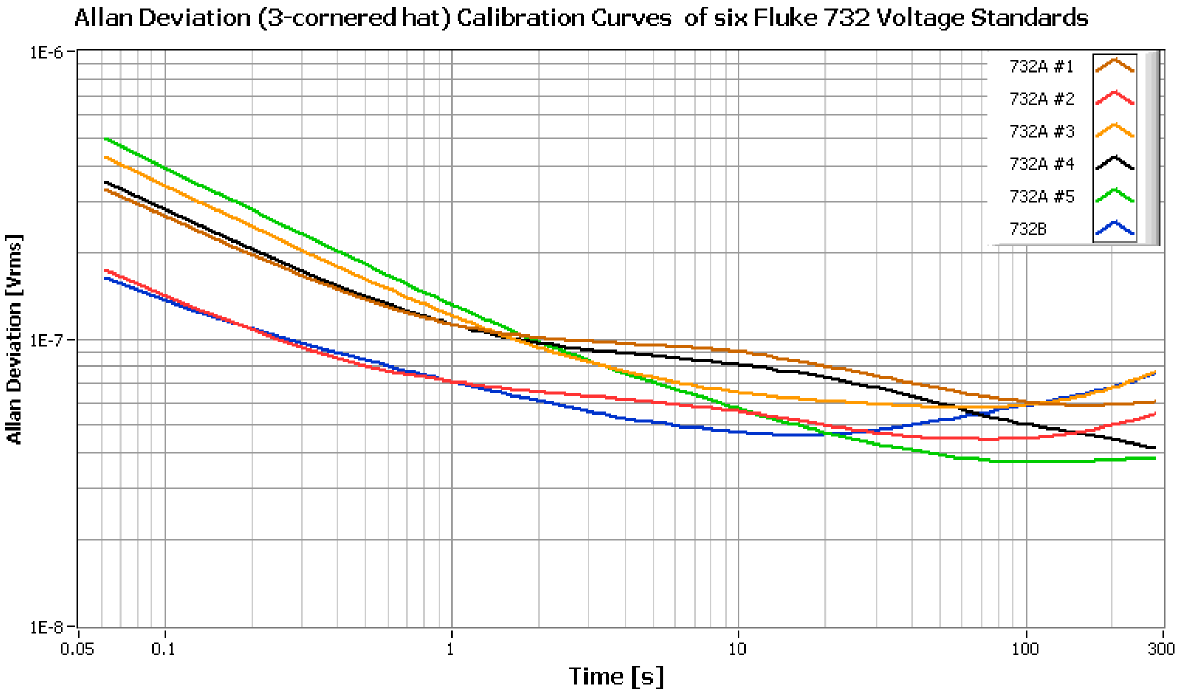

Allan Deviation (3-cornered hat) of Fluke 732A #4 S/N 4635005
Allan Deviation [Vrms]
1E-6
1E-7
1E-8
0.01
0.1
1
10
100
1000
10000
100000
Time [s]
124
134
145
14B
234
245
24B
345
34B
45B



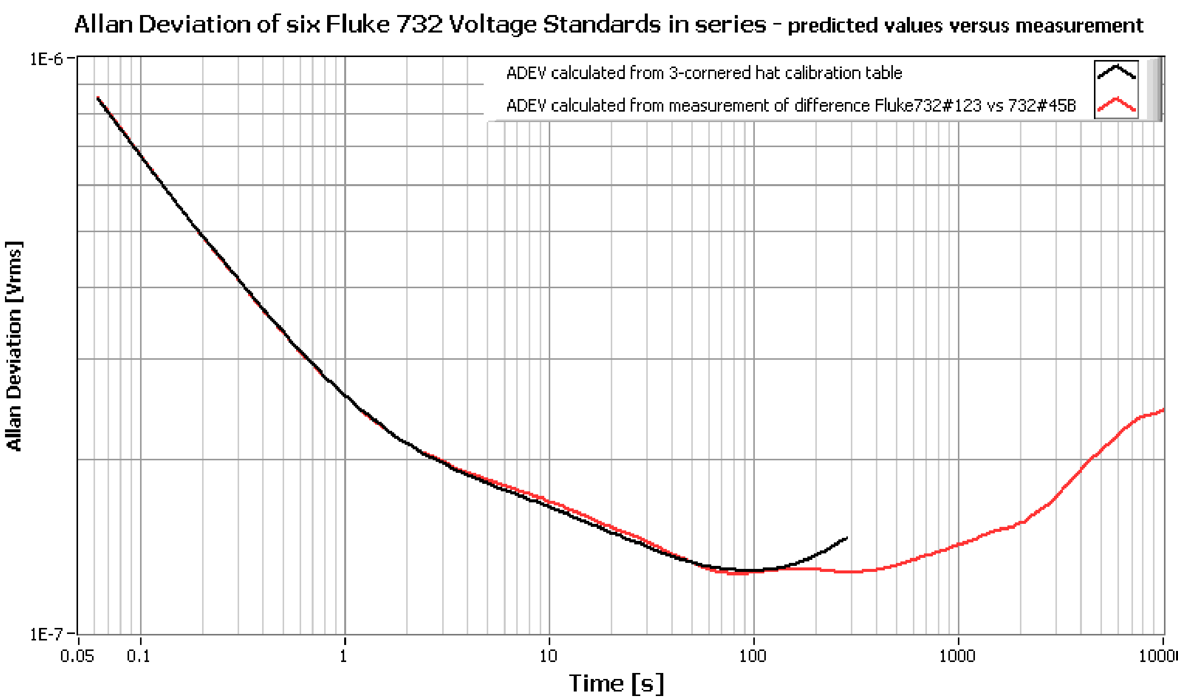


The names of the plots indicate which reference voltages were involved in the calculation of the curve.

E.g. 14B means: F732A#1, F732A#4 and F732B were used to calculate this curve.



These are my six calibration curves - the mean values of the ten solutions of each reference



Calculated noise (RMS of six references in series) versus real measurement

3-cornered hat calibration table F732B

# time	732B
0.062	1.628E-7
0.124	1.259E-7
0.186	1.109E-7
0.248	1.020E-7
0.310	9.598E-8
0.372	9.135E-8
0.434	8.769E-8
0.496	8.459E-8
0.558	8.201E-8
0.620	7.981E-8
0.682	7.793E-8
0.744	7.626E-8
0.806	7.477E-8
0.868	7.343E-8
0.930	7.224E-8
0.992	7.116E-8
1.054	7.017E-8
1.116	6.926E-8
1.178	6.842E-8
1.240	6.763E-8
1.302	6.688E-8
1.364	6.618E-8
1.488	6.490E-8

1.612	6.377E-8
1.736	6.274E-8
1.860	6.181E-8
1.984	6.095E-8
2.108	6.016E-8
2.232	5.942E-8
2.356	5.872E-8
2.480	5.808E-8
2.790	5.667E-8
3.100	5.550E-8
3.410	5.451E-8
3.720	5.367E-8
4.340	5.233E-8
4.960	5.134E-8
5.580	5.055E-8
6.200	4.986E-8
6.820	4.925E-8
7.440	4.872E-8
8.060	4.826E-8
8.680	4.786E-8
9.300	4.752E-8
9.920	4.722E-8
10.540	4.697E-8
11.160	4.677E-8
11.780	4.660E-8

12.400	4.646E-8
13.020	4.634E-8
13.640	4.625E-8
14.880	4.614E-8
16.120	4.611E-8
17.360	4.616E-8
18.600	4.624E-8
19.840	4.636E-8
21.080	4.652E-8
22.320	4.669E-8
23.560	4.687E-8
24.800	4.707E-8
27.900	4.764E-8
31.000	4.827E-8
34.100	4.889E-8
37.200	4.950E-8
43.400	5.071E-8
49.600	5.192E-8
55.800	5.304E-8
62.000	5.403E-8
68.200	5.492E-8
74.400	5.574E-8
80.600	5.650E-8
86.800	5.723E-8
93.000	5.792E-8

99.200	5.860E-8
105.400	5.925E-8
111.600	5.989E-8
117.800	6.052E-8
124.000	6.113E-8
130.200	6.173E-8
136.400	6.232E-8
148.800	6.347E-8
161.200	6.460E-8
173.600	6.570E-8
186.000	6.680E-8
198.400	6.790E-8
210.800	6.901E-8
223.200	7.016E-8
235.600	7.133E-8
248.000	7.253E-8
279.000	7.562E-8